FRACTIONAL DELAY OF EEG SIGNAL

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Abstract: The contribution compares various methods for the implementation of the EEG signal fractional delay block. The target of the application is to compensate the sampling instant time shift caused by the channel time multiplexing of the EEG signal AD conversion. Three methods to shift the signal are presented: linear and quadratic interpolation in the time domain as well as the DTFT application. Simple FIR filters are designed for the proposed methods and the best method is chosen for the EEG processing.

Keywords: EEG recording, fractional delay, surface filtering, polynomial interpolation

1 Introduction

EEG recording machines usually contains one AD converter which digitalises all the recorded EEG channels. The channels are converted sequentially, one after the other and thus time multiplexed. This brings problems with subsequent EEG processing because of the time shift between the sampling instants of the single channels. To present an example of such a problem let us recall the commonly used surface laplacian filter (see e.g. [11]). This filter should attenuate the EEG activity which is common to all the involved channels in order to improve the spatial resolution of the recorded signals. The time shift of the sampling instants of the EEG channels results in disturbing of this behaviour as will be shown later.

Suggested technique is useful even when the surface filtration is not used. A typical example is the work published in [2] in which the authors computed spatial spectrum from raw EEG without compensation. The introduced noise under such a conditions was a subject of [9]. The authors of [1], [5] or [8] might utilise our approach as well. Further application of our method is the EEG preprocessing for PCA/ICA transformation. ICA (see [3] or [4]) requires all processing signals sampled simultaneously.

The problem itself might be overcome in two ways:

1. using the EEG machine with the same numbers of AD converters as channels. This solution will work perfectly, its only drawback is the cost – if you sample 128 channels, you will need 128 AD converters.

2. post-filtering the data and time shifting them after recording. This approach avoids costly hardware and allows reasonable well time-shift compensation as will be discussed below.

The contribution is organised as follows: in the next section we will deal with the fractional delay realisations; then filter properties are discussed and improvement of the surface laplacian is presented in Section 3. Finally, Matlab implementation of the whole framework is presented and some conclusions are drawn.

2 Fractional Delay Realisations

A typical EEG recording machine contains only one AD converter used for all EEG channels (electrodes) digitalisation. The channels are sampled sequentially, one after the other – they are time multiplexed. This results in shifting of the sampling instants[1] and introduces a systematic error to the measurement. The importance of the error depends on the sampling frequency $f_s = \frac{1}{T_s}$, length of AD conversion period T_{AD} and on processing used afterwards.

The introduced noise might be compensated by means of suitable interpolation method. The following possibilities are obvious:

1. **linear interpolation** – proposed in [7]. The new sample $x[t]$ is computed from the neighbouring samples $x[t]$, $x[t-1]$ ($\delta = T_{AD}/T_s$, signal is sampled at the electrode with index s; the sampling instant is sT_{AD} delayed compared to the zeroth electrode sample time):

$$x'[t] = x[t] + x[t+1] - x[t](-s)\delta. \tag{1}$$

Required computational power is quite low for this approach – the number of operations is proportional to the overall number of samples $T - O(T)$.

[1] We are avoiding the term "jitter" through the text intentionally. Compared to aperture or clock jitter – see e.g. [6] – this process is not random in nature. The introduced error is systematic and deterministic one.

2. **higher order interpolation** – e.g. interpolation with the polynomial function of the second order:

$$x'[t] = (-s)^2\delta^2(\frac{x[t-1]-2x[t]+x[t+1]}{2}) + (-s)\delta\frac{x[t+1]-x[t-1]}{2} + x[t]. \qquad (2)$$

The number of operations is again proportional to the overall number of samples – $O(T)$.

3. **time shift in the spectral domain** – DTFT is applied, the phase spectrum is rotated and IDTFT is computed ([10]):

$$\begin{aligned} \mathbf{X} &= DTFT(\mathbf{x}) \\ \mathbf{X}[k] &= \mathbf{X}[k]e^{-j2\pi(-s)\delta k} \\ \mathbf{x}' &= IDTFT(\mathbf{X}) \end{aligned}$$

This approach perfectly suppress the sampling error influence on the filter output signal-to-noise ratio. The less desirable fact is higher computational complexity of the solution; for the vector of T samples processing at least $O(TlogT)$ operations are needed.

All the interpolations might also be viewed as filters. Their transfer functions (after some rearrangement and time shift in order to get causal systems) might be expressed as follows:

$$\begin{array}{ll} \text{linear} & \mathcal{H}_l(j\omega) = -s\delta + e^{-j\omega}(1+s\delta) \\ \text{quadratic} & \mathcal{H}_q(j\omega) = \frac{1}{2}(s^2\delta^2 - s\delta) + e^{-j\omega}(1-s\delta^2) + \frac{1}{2}e^{-j\omega 2}(s^2\delta^2 + s\delta) \\ \text{DTFT} & \mathcal{H}_D(j\omega) = e^{j\frac{2\pi n s\delta}{N}}. \end{array} \qquad (3)$$

All transfers are compared in Fig. 1. Obviously, all three filters have nearly identical phase characteristic and differ in the magnitude response. While the DTFT filter has $|\mathcal{H}_D(j\omega)|$ exactly equal to 1, the transfer function $|\mathcal{H}_q(j\omega)|$ of the quadratic interpolator is approximately constant and its value only slightly depends on the δ value. The $|\mathcal{H}_l(j\omega)|$ exhibits the worst behaviour of all.

3 Impact on the Surface Laplacian Filtration

The digitised EEG channels are filtered after recording. The purpose of this filtering is to remove the damping and mothing caused by the skull, scalp and cerebrospinal fluid on the sampled signals. Suitable surface filter is used for this purpose – usually one of the variants of laplacian ([11]). The frequency response of the surface filter has high-pass character in contrast to low-pass transfer function of the head and bone layers. Thus, under ideal conditions, all the attenuation of EEG inside the head is compensated in the subsequent surface filtration. However, this is often not the case – see [11]. One of the noise sources in the filtration process is related to the sampling time instant error mentioned above. The sampling error adds additive noise to the reference-free filtered output signal.

It is possible to compute the output error magnitude (common noise level) theoretically. At first, we have to introduce the following definitions:

- N – the number of electrodes located on the scalp
- $f_s = \frac{1}{T_s}$ – the sampling rate
- $V_i[t]$ – the potential under electrode with index i in time instant t
- V_i' – the resultant electrode potential (after filtering)
- c – the surface filter central electrode index; we compute the potential under this electrode
- S – the sequence of main electrode neighbours. S contains all indices of the surrounding electrodes used for the computation
- $k[i]$ – filtration coefficients – sequence ($k[i]$ is the coefficient corresponding to the electrode with the index i)
- M – the total number of electrodes used during the computation.

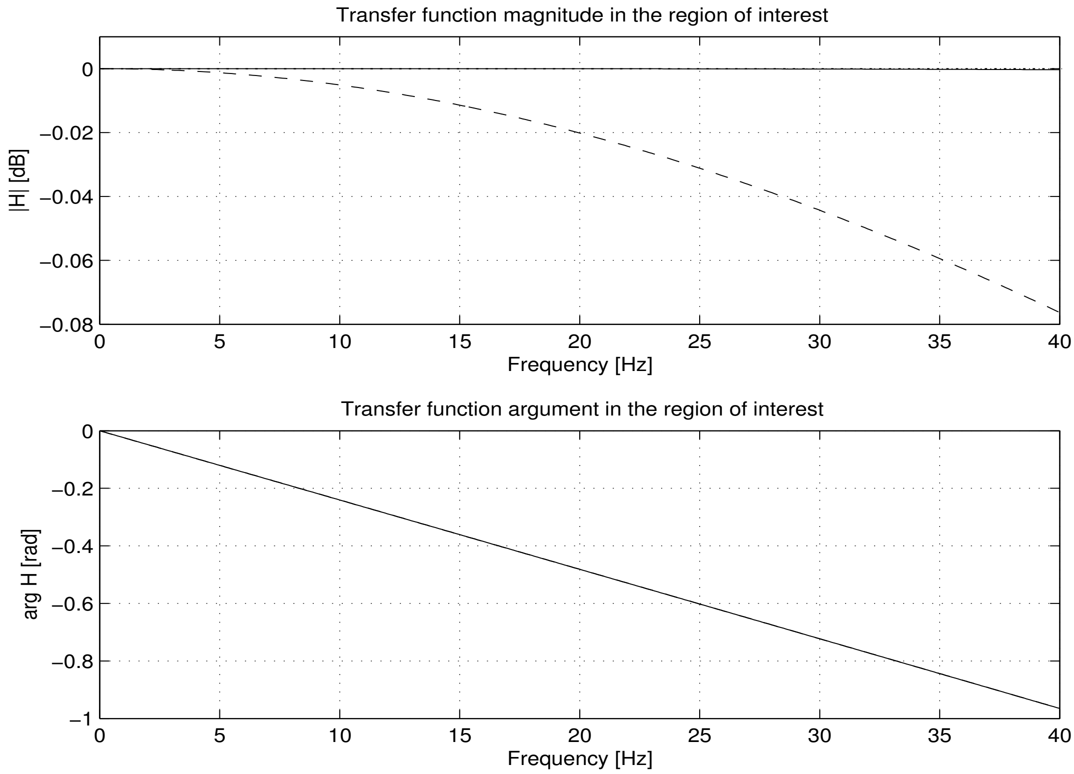

Figure 1: The transfer functions of the three interpolation techniques. The magnitude of the transfer is shown in the upper plot. Dashed line is for the linear, dotted for DFT, and solid for quadratic interpolation (they are nearly overlaid here). The linear interpolation exhibits the worst behaviour − the curve falls fast from 0dB compared to the more convenient quadratic interpolation behaviour and ideal DFT curve. The phase response of all three filters is linear, resulting in ADC introduced delay compensation (all the three responses are overlaid here and drawn as one solid). The pictures were computed for $\delta = 10T_{AD}$, our measurement configuration. Although the differences in the magnitude spectrum between the linear interpolation and the two other approaches are very small (less than 0.08dB) they are crucial and result in substantial differences in the quality of the results as will be shown below.

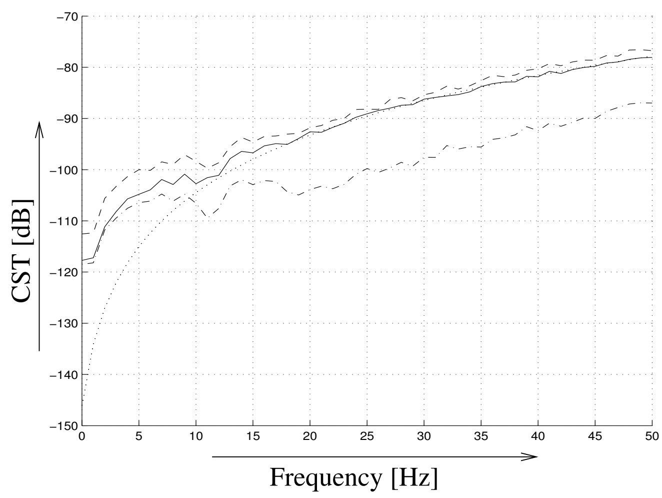

Figure 2: CST with SSL filter. CST is computed from (6) and drawn with dotted line. The solid line is CST computed with real EEG without interpolation, dashed line marks CST level with linear interpolation and dashed-dotted line with quadratic interpolation. The CST was obtained as the mean value of CSTs computed for one hundred different realisation of real input EEG at the SSL filter input. Obviously, the linear interpolation lowers the filtration quality. See discussion in the text.

The filtration can be generally described using introduced symbols

$$V_c'[t] = V_c[t] + \sum_{i=0}^{M-1} k[i] V_{S[n]}[t]. \tag{4}$$

Let's suppose that a harmonic signal with a generic phase shift φ and unity amplitude will be fed to all the filter inputs so as we can evaluate the common signal frequency response. This will give us the information on the filter common mode noise suppression behaviour. The sampling instant shift is taken into account as well. The resulting potential under electrode with index $S[i]$ is

$$V_{S[n]}[t] = \sin(\omega t T_s + \varphi + S[i] T_{AD} \omega), \tag{5}$$

where the last product $S[i]T_{AD}\omega$ represents the mentioned sampling instant shift. Now we will put (5) into the context of the (4) equation and after some rearrangement we get the following common signal transfer function:

$$|H(\omega)|_{dB} = 20log \left| 1 + 2 \sum_{\substack{i=1 \\ S[i]<c}}^{M} k[i] \cos((S[i] - c) T_{AD} \omega) \right| \tag{6}$$

Analysis of the equation (6) leads to the following conclusions: the CST grows with

- growing ratio of the ADC conversion time T_{AD} to the sample period $T_s = 1/f_s$
- growing number of electrodes used for recording
- CST is higher with higher common signal frequency.

The second notable thing is that we are dealing with surface correlated signals; the CST quantifies the amount of spatially correlated noise suppression.

The demonstrational example is computed for the experimental configuration used in our experiments:$f_s = 256Hz$, $T_{AD} = 7.81\mu sec$ (ADC conversion time), $N = 41$ electrodes. The electrodes are placed equidistantly in an orthogonal grid vertices above both sensomotoric areas of the experimental subject. Their spacing is chosen as 2.5cm (see e.g. [11]). The presented examples of CST were computed for spatial filter driven by the real EEG signal.

The shape of CST for the small surface laplacian filter (see [7]) after quadratic as well as linear interpolation of the real EEG is described in Fig. 2. It is noticeable that the theoretic (dotted) noise level computed according to formula (6) is in good agreement with the real noise level marked with solid line; the CST level gained with linear interpolation is higher than the level without any interpolation (the noise is less suppressed). The quadratic interpolation improves the results by about 10dB.

Further, the observed results are in compliance with the behaviour of the interpolating filter drawn in Fig. 1. The linear interpolator attains worse results than the quadratic one due to its far-from-constant magnitude characteristic.

4 Matlab Implementation

Our work is targeted to the design of the brain-computer interface (see e.g. [12] or [13]). A complete framework for experimenting with the EEG signal was implemented in Matlab, C++ and Bourne shell. The whole system covers the following parts of the processing:

EEG extraction a C++ program reads the files with recorded EEG and generates text files with EEG data for the subsequent Matlab processing.

delay compensation and surface filtration a Matlab script reads the extracted data as well as the coordinates of the electrodes on the experimental person's scalp, compensates the sampling instant shift and finally performs the surface filtration.

separation is done in Matlab, too. The input are the filtered EEG data and the position of the mental states in the EEG, the output are the single realisations of the appropriate mental states.

parameterisation generates parameterised realisations of the EEG for the following classification. Various parameters can be used – among others FFT lines, LPC coefficients or AR coefficients.

classification is performed with the help of the HTK toolkit – a suite of C++ programs implementing the Hidden Markov Model classifier.

results analysis is a set of scripts again written in Matlab which supports the analysis of the classification results. Average classification scores and various others statistical indicators are computed on the base of the classification outputs.

In addition, a lot of Matlab functions written for and targeted to EEG processing exist. Two examples of the performed tasks follow:

statistical analysis – sign, F, T or χ^2 tests applied on the EEG to reveal hidden patterns in the signal, display confidence interval, etc. The output of such a test might be e.g. the movement-related EEG pattern detection probability.

short-time power spectra analysis – computation of the time-development of the short-time power spectra, normalisation and further processing.

The system is still under development, its evolution reflects the current research needs. Currently, one of our students work on ICA functionality integration to the system (a third-party product, FastICA toolbox, will be used) to test the impact of the ICA denoising properties on the EEG classification.

5 Conclusion

The idea of interpolation was originally mentioned in [7]. However, the presented linear interpolation seems to be unsatisfying under some conditions (see Fig. 2). Thus we proposed the application of higher order interpolation or the DTFT application. The model for CST computation without the compensation was presented and compared with real system behaviour. While quadratic interpolation gives only an approximative common signal suppression, the Fourier transform compensates the error absolutely. The improved surface filtration is performed in two steps – first a delay correction is made and then the SSL filter is applied on the corrected channels.

The selection of the used countermeasures lies on the researcher doing the EEG measurement. The proper method shall be selected on the base of the used electrode configuration, EEG machine parameters and AD converter number of bits.

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